PHILOSOPHICAL TRANSACTIONS.

I. *An Experimental Determination of the Velocity of Sound.*

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A galvanic current passes from the batteries at the Royal Observatory, Cape of Good Hope, at 1 o'clock, Cape mean time. This current discharges a gun at the Castle, and through relays drops a time-ball at Port Elizabeth.

It appeared to me that a valuable determination of the velocity of sound might be obtained by measuring upon the chronograph of the Observatory the time between the sound reaching some point near the gun and that of its arrival at the Observatory. I thought also that it would be a point of interest to check, within the limits of our changes of temperature, the variations in the velocity of sound as dependent upon temperature, and to obtain some test of the applicability of the coefficient of expansion of dry air, as determined in cabinet experiments, to the mixture of air and water which would be the medium of the propagation of sound in our experiments.

There is only a single wire between the Observatory and Cape Town; some little difficulty was therefore experienced in making the necessary arrangements, without any interference with the 1 o'clock current to Port Elizabeth. I have adopted the following plan, which was brought into successful operation on 1871, February 27. It would, however, have been quite impossible for me to have had these experiments made, without an encroachment upon the time of the Observatory staff which could not have been sanctioned, had it not been for the assistance of J. Den, Esq., the acting manager of the Cape Telegraph Company. I am indebted to Mr. Den for the preparation of a good earth near the gun, for the assistance of one of the gentlemen attached to the telegraph office, Mr. Kirby, who has made all the observations at the Cape-Town end, and for a general superintendence of the arrangements in Cape Town. Mr. Kirby stands at a distance of 641 feet from the gun, near an earth whose connexion with the single main wire is broken at a tapping-piece which Mr. Kirby, at the time of the experiments, holds in his hand. A small battery is arranged at the Observatory with one pole to earth through the chronograph coil, and the other connected with the Cape-Town wire through a tapping-piece similar to that used by Mr. KIRBY. At 1 o'clock the observer at the Observatory (Mr. MANN) connects the local battery with the main line: this current is arranged so that it merely assists the main time-ball current. Mr. MANN holds down his tapping-piece until three seconds after 1 o'clock, and thus affords a connexion through the chronograph coils to register Mr. KIRBY's signal. When the current has passed the telegraph office in Cape Town, the connexion is broken at that office. Mr. KIRBY'S distance from the gun has been arranged so as to allow of this being done before the sound reaches his station. The line after the breaking of the connexion at Cape Town is complete except at Mr. KIRBY's tapping-piece. When the sound reaches Mr. KIRBY's station he completes the circuit, and his observation is registered on the Observatory chronograph. Mr. KIRBY then holds down his tapping-piece for half a minute, to make earth for the observer at the Observatory station. The connexion at the Observatory station is broken, as before stated, at three seconds after 1 o'clock. When the sound reaches the Observatory, about $13^{s}\cdot2$ after Mr. KIRBY's observation, the Observatory tapping-piece is again connected, and the time of the sound reaching this station recorded on the chronograph. Time-signals are then sent to check the loss of time of gun-fire, but not as bearing on the determination of the velocity of sound, the results for which are quite independent of any loss of time at the gun, or of any errors of rate except that of the chronograph between seconds of the transit-clock and of the transit-clock for about 13^{s}.

The observations have been made on all the days since February 27 upon which Mr. KIRBY's services were available without any interference with his regular duties. The observations will be found in Table I.

The results have been corrected for the effects of the motion of the air upon the difference in time between the sound reaching Mr. KIRBY's station and its reaching the Observatory, with velocities of the wind found from a set of ROBINSON's cups.

To reduce the equations of condition to a linear form corrections have been applied for the second and third terms of the expansion of $\sqrt{1+\alpha\theta}$, where α is the coefficient of the elasticity of air under a constant volume for a degree FAHRENHEIT of temperature, and θ is the excess of the temperature at the time of the experiment above 32°. The observed differences have also been diminished by $-0^{s}\cdot09$ for the effects of personal equation between Mr. MANN and Mr. KIRBY under the circumstances of these observations.

This personal equation has been found as follows:—A gun was fired at such a distance from the Observatory that the sound was heard with about the same degree of distinctness as the ordinary time-ball gun at the Castle. This was at a distance of 1483 feet from the Observatory. Mr. KIRBY was placed at a distance of 162 feet from the gun. From previous determinations of the velocity of sound, or from the first approximate result of the present experiments, we can compute with great accuracy the difference in time, at the temperature of the air at the time of observation, of the sound reaching

Mr. KIRBY near the gun and Mr. MANN at the Observatory. The computed difference was $1^{s}\cdot177$; but the observed difference, with the same observers and with the same tapping-pieces as those used in the principal experiments, was $1^{s}\cdot265$: this was the result from twelve accordant observations. The difference $0^{s}\cdot09$ has been applied to all the observed results.

This correction depends more upon want of sensibility in picking up and recognizing faint sounds, than upon mere habit of making contacts. When the observers were reversed and Mr. KIRBY stationed at the Observatory and Mr. MANN near the gun, the observed difference appeared still too large, but in this case by $0^{s}\cdot20$. It is clear that such personal equations are not eliminated by an interchange of observers nor by return signals.

The equations of condition appear in Table II. The times given are those observed corrected for the motion of the air, the second and third terms of the effects of temperature above 32°, and for personal equations. In these equations

$$x=\frac{14808\cdot5\text{ feet}}{V},\ y=\frac{\alpha x}{2},\ V=\text{velocity of sound at }32^{\circ}.$$

The solution of these equations gives

$$V=1090\cdot6\text{ feet per second,}$$
$$\alpha=0\cdot0019.$$

REGNAULT'S value of α is 0·0020.

The agreement between the value of α deduced from these experiments and REGNAULT'S value is so close that the difference between these values would scarcely be appreciable within the limits of variation of temperature in our experiments. The whole of the results have been given equal weights. It has not appeared necessary to attempt any discrimination between the results in the present paper. There appears, indeed, but little difference between the residuals as dependent upon the corrections for the motion of air. I have grouped the residuals into two classes according to the dampness of the air; but there appears no difference in the velocity, as dependent upon dampness, appreciable within the limits of these experiments, either when referred to tension or humidity. The mean residual for each group nearly vanishes. The whole of the measurements of the distances involved have been made by Mr. MANN. The observations of the regular series from February 27 have been made by Mr. KIRBY at the Cape-Town end, near the gun, and by Mr. MANN at the Observatory. The arrangements for the experiments, galvanic and otherwise, the determination of the personal equations, and the discussion of the results have been made by myself.

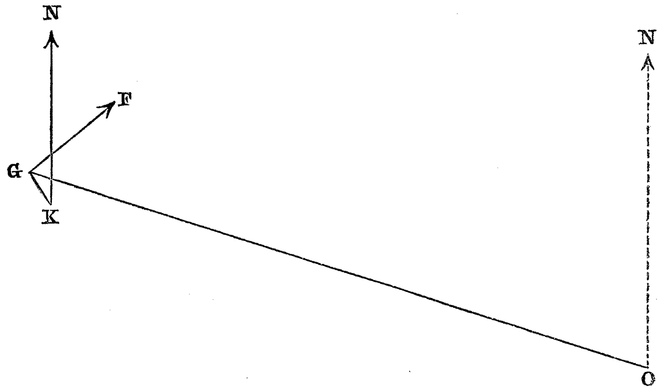
Table I.—Experiments for the Determination of the Velocity of Sound.

No of Expt.	Date.	Thermometers.		Barometer.	Wind.		Observed diff.	Correction for motion of air.	Diff. corrected for motion of air.	Residuals computed − observed.
		Dry.	Wet.		Direction, Az.	Velocity, in miles per hour.				
	1871.	°	°	in.	°		s.	s.	s.	s.
1.	Feb. 27	80·4	66·6	30·036	188	7	13·10	−0·01	13·09	−0·01
2.	Feb. 28	77·6	69·8	29·986	158	9	13·32	−0·09	13·23	−0·11
3.	Mar. 1	69·0	61·1	30·222	173	25	13·53	−0·14	13·39	−0·17
4.	Mar. 2	79·4	63·0	30·193	177	18	13·22	−0·08	13·14	−0·04
5.	Mar. 3	88·1	66·7	30·000	326	4	12·88	+0·05	12·93	+0·07
6.	Mar. 4	75·3	67·4	30·067	186	19	13·29	−0·04	13·25	−0·10
7.	Mar. 6	71·6	61·8	30·152	175	32	13·46	−0·17	13·29	−0·10
8.	Mar. 7	82·1	65·2	30·084	163	9	13·05	−0·08	12·97	+0·10
9.	Mar. 9	74·4	67·5	30·130	315	9	12·81	+0·13	12·94	+0·21
10.	Mar. 23	69·0	57·3	30·194	171	12	13·26	−0·08	13·18	+0·04
11.	Mar. 24	71·9	61·0	30·229	275	7	13·15	+0·12	13·27	−0·09
12.	Mar. 27	82·1	64·5	29·964	315	5	12·95	+0·07	13·02	+0·05
13.	Mar. 30	58·9	52·4	30·128	213	10	13·16	+0·06	13·22	+0·11
14.	Mar. 31	65·7	59·5	30·286	275	5	13·32	+0·09	13·41	−0·16
15.	April 1	70·2	63·7	30·176	180	18	13·36	−0·07	13·29	−0·09
16.	April 4	74·7	63·3	29·929	328	7	12·84	+0·09	12·93	+0·23
17.	April 5	68·0	63·5	29·988	302	10	13·02	+0·16	13·18	+0·05
18.	April 6	69·0	62·6	30·218	281	9	13·13	+0·16	13·29	−0·07
19.	April 11	80·0	65·4	30·150	152	9	13·19	−0·10	13·09	0·00
20.	April 12	77·0	66·2	30·146	158	14	13·27	−0·14	13·13	0·00
21.	April 13	71·4	63·7	30·050	167	15	13·38	−0·11	13·27	−0·08
22.	April 15	70·5	63·8	30·216	0	1	12·87	0·00	12·87	+0·33
23.	April 17	64·1	60·1	30·060	318	5	13·44	+0·08	13·52	−0·25
24.	April 25	58·4	54·1	30·074	121	3	13·42	−0·05	13·37	−0·03
25.	April 27	64·0	58·0	30·336	270	2	13·19	+0·03	13·22	+0·06
26.	April 28	62·5	55·6	30·280	315	9	12·82	+0·13	12·95	+0·35
27.	April 29	67·8	55·3	30·262	158	9	13·43	−0·09	13·34	−0·10
28.	May 1	83·0	60·8	30·056	282	11	12·92	+0·19	13·11	−0·05
29.	May 6	65·0	60·3	30·037	343	2	13·64	+0·02	13·66	−0·40
30.	May 8	62·3	59·1	30·028	321	9	13·22	+0·13	13·35	−0·05
31.	May 9	62·6	59·5	29·942	357	8	13·19	+0·04	13·23	+0·06
32.	May 12	66·4	58·9	30·006	279	11	13·09	+0·19	13·28	−0·04
33.	May 16	61·2	59·3	29·924	321	7	12·95	+0·10	13·05	+0·26
34.	May 20	61·2	55·2	30·136	343	6	13·16	+0·05	13·21	+0·10
35.	May 23	68·2	55·6	30·128	315	4	13·24	+0·06	13·30	−0·07
36.	June 6	55·6	50·4	30·050	264	14	12·99	+0·23	13·22	+0·15
37.	June 9	61·7	56·1	30·176	304	11	13·15	+0·18	13·33	−0·02
38.	June 26	53·4	51·2	30·270	197	15	13·50	+0·02	13·52	−0·12

TABLE II.—Equations of Condition.

	s
1.	$x - 48{\cdot}4\,y = 12{\cdot}96$
2.	$x - 45{\cdot}6\,y = 13{\cdot}10$
3.	$x - 37{\cdot}0\,y = 13{\cdot}27$
4.	$x - 47{\cdot}4\,y = 13{\cdot}01$
5.	$x - 56{\cdot}1\,y = 12{\cdot}78$
6.	$x - 43{\cdot}3\,y = 13{\cdot}12$
7.	$x - 39{\cdot}6\,y = 13{\cdot}17$
8.	$x - 50{\cdot}1\,y = 12{\cdot}83$
9.	$x - 42{\cdot}4\,y = 12{\cdot}82$
10.	$x - 37{\cdot}0\,y = 13{\cdot}06$
11.	$x - 39{\cdot}9\,y = 13{\cdot}15$
12.	$x - 50{\cdot}1\,y = 12{\cdot}88$
13.	$x - 26{\cdot}9\,y = 13{\cdot}12$
14.	$x - 33{\cdot}7\,y = 13{\cdot}30$
15.	$x - 38{\cdot}2\,y = 13{\cdot}17$
16.	$x - 42{\cdot}7\,y = 12{\cdot}80$
17.	$x - 36{\cdot}0\,y = 13{\cdot}06$
18.	$x - 37{\cdot}0\,y = 13{\cdot}17$
19.	$x - 48{\cdot}0\,y = 12{\cdot}96$
20.	$x - 45{\cdot}0\,y = 13{\cdot}00$
21.	$x - 39{\cdot}4\,y = 13{\cdot}15$
22.	$x - 38{\cdot}5\,y = 12{\cdot}75$
23.	$x - 32{\cdot}1\,y = 13{\cdot}41$
24.	$x - 26{\cdot}4\,y = 13{\cdot}27$
25.	$x - 32{\cdot}0\,y = 13{\cdot}11$
26.	$x - 30{\cdot}5\,y = 12{\cdot}84$
27.	$x - 35{\cdot}8\,y = 13{\cdot}22$
28.	$x - 51{\cdot}0\,y = 12{\cdot}97$
29.	$x - 33{\cdot}0\,y = 13{\cdot}55$
30.	$x - 30{\cdot}3\,y = 13{\cdot}24$
31.	$x - 30{\cdot}6\,y = 13{\cdot}12$
32.	$x - 34{\cdot}4\,y = 13{\cdot}17$
33.	$x - 29{\cdot}2\,y = 12{\cdot}94$
34.	$x - 29{\cdot}2\,y = 13{\cdot}10$
35.	$x - 36{\cdot}2\,y = 13{\cdot}18$
36.	$x - 23{\cdot}6\,y = 13{\cdot}12$
37.	$x - 29{\cdot}7\,y = 13{\cdot}22$
38.	$x - 21{\cdot}4\,y = 13{\cdot}42$

From which we find $x=13{\cdot}578$ and $y=0{\cdot}0129$.



O is the position of the observer at the Observatory.

K is the position of the observer near the gun G.

F indicates the direction of the gun.

N, N point north.

G O=15449 feet; G K=640·5 feet.

Angle O G K=36° 48′; G O N=76° 2′.